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ELECTROMAGNETIC WAVE PROPAGATION IN PULSAR MAGNETOSPHERES

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ABSTRACT

A new nonlinear electromagnetic wave mode in a magnetized plasma is predicted. Its existence depends on the interaction of an intense circularly polarized electromagnetic wave with a plasma, where quantum electrodynamical photon–photon scattering is taken into account. This scattering gives rise to a new coupling between the matter and the radiation. Specifically, we consider an electron–positron plasma, and show that the propagation of the new mode is admitted. It could be of significance in pulsar magnetospheres, and result in energy transport between the pulsar poles.

Subject headings: Plasmas — pulsars: general — stars: neutron — waves

Astrophysical environments can be most violent and energetic. Physics considered 'exotic' in Earth based laboratory applications can be common throughout our Universe, and sometimes even vital for the existence of certain observed phenomena. Pulsars, surrounded by strong magnetic fields, are most prolific sources of exotic physics. Quantum electrodynamics (QED) is an indispensable explanatory model for much of the observed pulsar phenomena. Scattering of photons off photons is predicted by QED, and it can be a prominent component of pulsar physics, since pulsars offer the necessary energy scales for such scattering to occur. Related to the scattering of photons is the concept of photon splitting in strong magnetic fields (Adler 1971). It has been suggested that such effects could be important in explaining the radio silence of magnetars (Kouveliotou 1998; Baring & Harding 2001). In the present Letter we will point out the existence of a new electromagnetic wave that may exist in pulsar magnetospheres, due to the interaction of photons with the quantum vacuum. A discussion of the properties of this electromagnetic wave using parameters relevant to strongly magnetized pulsars will be given.

The weak field theory of photon–photon scattering can be formulated in terms of the effective Lagrangian density

$$\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_{\rm HE}, \tag{1}$$

where $\mathscr{L}_0 = -\frac{1}{4}\epsilon_0 F_{ab}F^{ab} = \frac{1}{2}\epsilon_0(\mathbf{E}^2 - c^2\mathbf{B}^2)$ is the classical free field Lagrangian, and

$$\mathscr{L}_{\rm HE} = \kappa\epsilon_0^2\left[4\left(\tfrac{1}{4}F_{ab}F^{ab}\right)^2 + 7\left(\tfrac{1}{4}F_{ab}\widehat{F}^{ab}\right)^2\right], \tag{2}$$

is the Heisenberg–Euler correction (Heisenberg & Euler 1936; Schwinger 1951), where $\widehat{F}_{ab} = \frac{1}{2}\epsilon_{abcd}F^{cd}$, and $\frac{1}{4}\widehat{F}_{ab}F^{ab} = -c\mathbf{E}\cdot\mathbf{B}$. Here $\kappa \equiv 2\alpha^2\hbar^3/45m_e^4c^5 \approx 1.63\times10^{-30}\,\mathrm{ms^2/kg}$, α is the fine-structure constant, $\hbar$ is the Planck constant, m_e is the electron mass, and c is the speed of light in vacuum. With $F_{ab} = \partial_a A_b - \partial_b A_a$, A^b being the four-potential, we obtain, from the Euler–Lagrange equations, the field equations $\partial_b[\partial\mathscr{L}/\partial F_{ab}] = 0$, i.e. (see, e.g. Shukla *et al.* 2004)

$$\partial_b F^{ab} = 2\epsilon_0\kappa\partial_b\left[(F_{cd}F^{cd})F^{ab} + \tfrac{7}{4}(F_{cd}\widehat{F}^{cd})\widehat{F}^{ab}\right] + \mu_0 j^a, \tag{3}$$

where j^a is the four current.

For a circularly polarized wave $\mathbf{E}_0 = E_0(\hat{\mathbf{x}}\pm i\hat{\mathbf{y}})\exp(i\mathbf{k}\cdot\mathbf{x} - i\omega t)$ propagating along a constant magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$, the invariants satisfy

$$F_{cd}F^{cd} = -2E_0^2\left(1 - \frac{k^2c^2}{\omega^2}\right) + 2c^2B_0^2 \text{ and } F_{cd}\widehat{F}^{cd} = 0, \tag{4}$$

where k is the wave number and ω the frequency of the circularly polarized electromagnetic wave. Thus, Eq. (3) can be written as

$$\Box A^a = -4\epsilon_0\kappa\left[E_0^2\left(1 - \frac{k^2c^2}{\omega^2}\right) - c^2B_0^2\right]\Box A^a + \mu_0 j^a, \tag{5}$$

in the Lorentz gauge, and $\Box = \partial_a\partial^a$. For circularly polarized electromagnetic waves propagating in a magnetized cold multicomponent plasma, the four current can be 'absorbed' in the wave operator on the left-hand side by the replacement

$$\Box \to -D(\omega, k), \tag{6}$$

where D is the plasma dispersion function, given by (see, e.g. Stenflo (1976) and Stenflo & Tsintsadze (1979))

$$D(\omega, k) = k^2c^2 - \omega^2 + \sum_j \frac{\omega\omega_{pj}^2}{\omega\gamma_j \pm \omega_{cj}}, \tag{7}$$

where the sum is over the plasma particle species j,

$$\omega_{cj} = \frac{q_jB_0}{m_{0j}} \quad \text{and} \quad \omega_{pj} = \left(\frac{n_{0j}q_j^2}{\epsilon_0 m_{0j}}\right)^{1/2}, \tag{8}$$

is the gyrofrequency and plasma frequency, respectively, and

$$\gamma_j = (1 + \nu_j^2)^{1/2}, \tag{9}$$

is the the gamma factor of species j, with ν_j satisfying

$$\nu_j^2 = \left(\frac{eE_0}{cm_{0j}}\right)^2\frac{1 + \nu_j^2}{[\omega(1 + \nu_j^2)^{1/2} \pm \omega_{cj}]^2}. \tag{10}$$

Here n_{0j} denotes particle density in the laboratory frame and m_{0j} particle rest mass.

Introducing the Schwinger critical field $E_S = m_e^2c^3/e\hbar \sim 10^{18}$ V/m, the dispersion relation, obtained from Eq. (5), reads

$$D(\omega,k) = \frac{4\alpha}{45\pi}(\omega^2 - k^2c^2) \times \left[\left(\frac{E_0}{E_S}\right)^2 \frac{\omega^2 - k^2c^2}{\omega^2} - \left(\frac{cB_0}{E_S}\right)^2\right]. \quad (11)$$

We note that as the plasma density goes to zero, the effect due to photon–photon scattering, as given by the right-hand side of Eq. (11), vanishes, since then $\omega^2 - k^2c^2 = 0$

Next, we focus on mode propagation in an ultra-relativistic electron–positron plasma ($\gamma_e \gg 1$), where the two species have the same number density n_0. Then Eq. (11) gives

$$k^2c^2 - \omega^2 \pm \frac{\omega\omega_{pe}^2}{\omega_E} = \frac{4\alpha}{45\pi}\left[\left(\frac{E_0}{E_S}\right)^2 \frac{\omega^2 - k^2c^2}{\omega^2} - \left(\frac{cB_0}{E_S}\right)^2\right](\omega^2 - k^2c^2). \quad (12)$$

Following Stenflo & Tsintsadze (1979), we have defined $\omega_E = eE_0/cm_{0e}$.

Looking for low-frequency modes, we now use the approximation $\omega \ll kc$, at which Eq. (12) gives

$$\frac{k^2c^2}{\omega^2} \approx \frac{4\alpha}{45\pi}\left[\left(\frac{E_0}{E_S}\right)^2 \frac{k^2c^2}{\omega^2} + \left(\frac{cB_0}{E_S}\right)^2\right]\frac{k^2c^2}{\omega^2} \mp \frac{\omega_{pe}^2}{\omega\omega_E}. \quad (13)$$

It is sometimes advantageous to use the relation $\omega_E = \omega_e(E_0/E_S)$, where $\omega_e = m_ec^2/\hbar$ is the Compton frequency, to write Eq. (13) as

$$\frac{k^2c^2}{\omega^2} \approx \frac{4\alpha}{45\pi}\left[\left(\frac{E_0}{E_S}\right)^2 \frac{k^2c^2}{\omega^2} + \left(\frac{cB_0}{E_S}\right)^2\right]\frac{k^2c^2}{\omega^2} \mp \frac{\omega_{pe}^2}{\omega\omega_e}\frac{E_S}{E_0}. \quad (14)$$

Using the dispersion relation (13) the group velocity $v_g \equiv d\omega/dk$ is

$$v_g = \frac{1 - \frac{4\alpha}{45\pi}\left(\frac{cB_0}{E_S}\right)^2 \pm \frac{2v_p}{kc^2}\frac{\omega_{pe}^2}{\omega_E}}{1 - \frac{4\alpha}{45\pi}\left(\frac{cB_0}{E_S}\right)^2 \pm \frac{3v_p}{2kc^2}\frac{\omega_{pe}^2}{\omega_E}} v_p, \quad (15)$$

where $v_p \equiv \omega/k$ is the phase velocity.

Pulsar magnetospheres exhibit extreme field strengths in a highly energetic pair plasma. Ordinary neutron stars have surface magnetic field strengths of the order of $10^6 - 10^9$ T, while magnetars can reach $10^{10} - 10^{11}$ T (Kouveliotou 1998), coming close to, or even surpassing, energy densities $\epsilon_0 E_S^2$ corresponding to the Schwinger limit. Such strong fields will make the vacuum fully nonlinear, due to the excitation of virtual pairs. Photon splitting can therefore play a significant role in these extreme systems (Harding 1991; Baring & Harding 2001). The emission of short wavelength photons due to the acceleration of plasma particles close to the polar caps results in production of electrons and positrons as the photons propagate through the pulsar intense magnetic field (Beskin 1993). Given the Goldreich–Julian density $n_{GJ} = 7 \times 10^{15}(0.1\,\mathrm{s}/P)(B/10^8\,\mathrm{T})\ \mathrm{m}^{-3}$, where P is the pulsar period and B the pulsar magnetic field, the pair plasma density is expected to satisfy $n_0 = Mn_{GJ}$, where M is the multiplicity (Beskin 1993; Luo *et al.* 2002). Moderate estimates give $M = 10$ (Luo *et al.* 2002). Thus, the density in a pulsar pair plasma can be of the order $10^{18}\ \mathrm{m}^{-3}$. The plasma experiences a relativistic factor $\sim 10^2 - 10^3$ (Asseo 2003). On the other hand, the primary beam will have $n_0 \sim n_{GJ}$ and $\gamma \sim 10^6 - 10^7$ (Asseo 2003).

For background fields strengths in the lower range given above (corresponding to pulsars rather than magnetars), $cB_0 \ll E_S$, and we therefore drop the term proportional to B_0^2 in Eq. (14). Next, using the normalized quantities $\Omega = \omega\omega_e/\omega_{pe}^2$, $K = (4\alpha/45\pi)^{-1/2}kc\omega_e/\omega_{pe}^2$ and $\mathcal{E} = (4\alpha/45\pi)E_0/E_S$, the dispersion relation (14) reads

$$\Omega^2 = \mathcal{E}^2K^2 \mp \frac{\Omega^3}{\mathcal{E}K^2}. \quad (16)$$

This dispersion relation describe three different modes, two with + polarization and one with − polarization. The normalized frequency as a function of K and $\mathcal{E}$ is shown in Fig. 1. We note that for $K \ll 1$, the dispersion relation (16) agrees with that of Stenflo &Tsintsadze (1979), whereas in the opposite limit $K \gg 1$, the QED term in (16) is dominating. For the given density, the latter regime applies, except for extremely long wavelengths ($> 10^8$ m), and thus we note that QED effects are highly relevant for the propagation of these modes in the pulsar environment. For small K there is only one mode, but as seen from the second and third panels of Fig. 1, two new modes appear for $K \gtrsim 2.6$. Thus for large K, applicable in the pulsar environment, there are three low-frequency modes ($\omega \ll kc$) that depend on nonlinear QED effects for their existence. Using $cB_0 \ll E_S$, the expression (15) for the group velocity becomes

$$\frac{d\Omega}{dK} = \frac{\Omega \pm 2\Omega^2/\mathcal{E}K^2}{\Omega \pm 3\Omega^2/2\mathcal{E}K^2}\frac{\Omega}{K}, \quad (17)$$

and thus we see that the propagation speed depends nonlinearly on the plasma parameters. We suggest that the three new modes presented above can contribute to an understanding of the very complicated energy transport phenomena taking place in the accretion discs of pulsars.

In summary, we have reported the existence of a new electromagnetic wave in pulsar magnetospheres. The dispersion relation of the wave has been presented, and analysed using relevant astrophysical parameters. Applications to pulsar magnetoplasmas have been pointed out.

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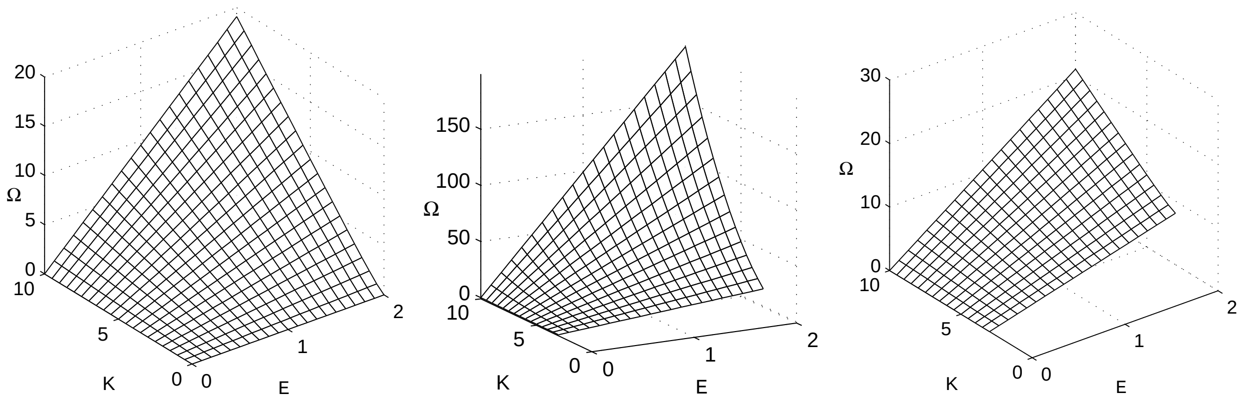

FIG. 1.— Dispersion surfaces $\Omega = \Omega(K, \mathcal{E})$ as given by Eq. (16). The first panel corresponds to the $-$ sign in Eq. (16), and exists for all K and $\mathcal{E}$. The second panel shows the fast $+$ polarized mode, which exists for $K \gtrsim 2.6$. The third and final panels depict the slow $+$ polarized mode, also for $K \gtrsim 2.6$.

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